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MODELING OF THE DYNAMICS OF THE HOMOGENEOUS TURBULENCE OF STEADILY STRATIFIED MEDIA

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Numerical modeling of the dynamics of the homogeneous turbulence of a steadily stratified liquid has been accomplished in the presence of vertical constant density gradient on the basis of a $\overline{u_i u_j}$—ϵ_u—$u_i \rho$—ϵ_ρ *second-order model, right up to the final stage of degeneration. The structure of relict turbulence has been investigated for values of the molecular Prandtl numbers corresponding to air and a sea wave.*

INTRODUCTION

Laws governing the dynamics (evolution) of the homogeneous turbulence of a steadily stratified liquid are of great theoretical and practical interest. From the theoretical point of view, this can be explained by the possibility of a more detailed (as compared to the shear turbulence of general form) investigation of the role of the density gradient as a turbulence generator in an actively stratified medium. From the practical viewpoint, homogeneous turbulence of a steadily stratified liquid is a realistic enough model of the upper atmosphere or the thermohalocline region in the ocean.

In order to elucidate fundamental specific features of the evolution of the steadily stratified homogeneous turbulence, in recent years experimental investigations were carried out both for a liquid (salt solution) and for air. In the work of T. D. Dickey and G. L. Mellor [1], which, evidently, is the first, carried out on the level of statistical parameters of experimental study of turbulence in a steadily stratified liquid, they have discovered a wavelike variation of the dispersion of vertical velocity fluctuations in the course of attaining a definite value of the dimensionless time Nt, which, by assumption, is indicative of the transfer of turbulent fluctuations to the field of internal waves. Besides, at the same value of Nt, a stepwise decrease in the rate of dissipation of the kinetic energy ε_u of disturbances has been discovered, confirming the existence of the transfer to the weakly dissipative wave process (in subsequent experiments the indicated behavior of ε_u has not been recorded). A wavelike behavior of the energy of transverse velocity fluctuations has been confirmed for the first time in the work of Riley et al. [2], who carried out a numerical study of the evolution of the homogeneous turbulence in the case of the steady stratification of a medium. At the same time, however, a stepwise change in the parameter ε_u has not been established, which testifies to the deficiency of the predominant contribution of internal waves to the velocity field, at least for the investigated range of Nt.

In the experimental works (more detailed than [1]) of Stillinger et al. [3] and Itsweire et al. [4] a great number of "energy" characteristics of the velocity and density fields, various characteristic length scales associated with the velocity and density fields, and the transverse turbulent density flux have been investigated. The latter parameter has been found to be extremely important for interpretating specific features attributed to the role of gravitation in the evolution of homogeneous turbulence. It has been shown that the parameter considered decreases to zero (the conventional collapse of turbulence) and then, with increase in Nt, acquires negative values, i.e., a countergradient density transfer is observed. Whereas in early investigations it has been assumed that the condition $\overline{u_2 \rho} = 0$ denotes a complete suppression of velocity vertical fluctuations and transition to the so-called two-dimensional relict turbulence, in [3, 4] they have demonstrated that gravitation slows down (as compared to the case

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of passive stratification) the degeneration of vertical fluctuations, i.e., after the "collapse" point they remain even still more significant than in the absence of the gravitation effect. In the indicated references it was shown that after the point at which $\overline{u_2 \rho} = 0$, a wavelike variation of all measured parameters is observed, and at the same time in the energy of vertical fluctuations a "break" in the degeneration law has been observed: the values of $\overline{u_2^2}$, averaged over the amplitude of fluctuations, degenerate after the "collapse" point in a self-similar way, just as before the "collapse" point, but with a smaller exponent in the power law (all the above refers, of course, to comparatively large values of turbulent Reynolds and Peclet numbers, corresponding to the indicated experiments).

In the most detailed experimental work of Lienhard and Van Atta [5], conducted for temperature-stratified air, the concept of homogeneous turbulence of a steadily stratified medium was developed as an intrinsically two-scale process, in which large-scale vortices are controlled by buoyancy forces, and small-scale ones - by viscosity. In view of this concept, it is shown that the condition $\overline{u_2 \rho} = 0$ does not imply the disappearance of active turbulent mixing, but is evidence of an important regularity of the stratified liquid turbulence: at a certain value of the "phase" Nt, the transverse mass flows associated with large and small vortices are equal in magnitude and opposite in sign. The change of the sign of the mass flux (countergradient transfer) is attributed especially to large-scale turbulent motion and is a consequence of restratification, i.e., the motion of large vortices "brought" by the turbulence from the region of higher density to the region of lower density, under the effect of gravity to the state of equilibrium, and further, to the region of higher density. As concerns the mass flux associated with small vortices, it is naturally directed at any moment of time.

Thus, the scales at which a turbulent mixing occurs and the scales at which the generation of internal waves occurs, revealing itself in the sign change of the mass flux, are significantly spread over the spectrum of scales. As concerns the mechanism of energy transfer from a relatively small-scale turbulence to a large-scale wave motion, at present it is not fairly clear. To elucidate this extremely important problem, a number of trials have been undertaken of direct numerical modeling of the homogeneous turbulence evolution of a stratified liquid. The first attempt was made by Riley et al. [2]. It has been shown by the authors of this work that, although stratification does increase the rate of dissipation degeneration of the kinetic energy of disturbances, this parameter, however, remains large enough so that the turbulence energy could be considered to be completely converted to the energy of internal waves. Thus, it follows from this work that, at least in the studied range of the dimensionless time Nt, internal gravitational waves coexist with the turbulence proper as with a purely stochastic field.

A detailed numerical investigation of stochastic and wave modes in the turbulence of a stratified medium has been performed in the recent work of Metais and Herring [6].

The results obtained in the numerical investigations indicated above show that development of homogeneous turbulence in a steadily stratified medium has been studied thoroughly enough for moderate values of turbulent Prandtl and Peclet numbers. In real situations present in the atmosphere and ocean, the conditions of small values of R_λ and P_λ (atmosphere) and small values of R_λ and moderate or even large values of P_λ (thermohalocline region of the ocean) are realized. Therefore, if the references stated above have a certain relation to the atmosphere, they have no relation at all to the ocean, since the case $P_\lambda \gg R_\lambda$ is as yet inaccessible for direct numerical modeling. Besides, for direct numerical modeling, large values of the evolution time, at which $R_\lambda \to 0$, are as yet also inaccessible.

In the present work an attempt has been made to apply the second-order model developed by the author of [7] to the study of the dynamics of the steadily stratified turbulence of a Boussinesq liquid. The following problems have been stated:

- the study of the dynamics of a relatively strong turbulence at various values of the total Froude number Fr = NM/U;

- the study of the effect of the molecular Prandtl number on the evolution (degeneration) of a moderately strong stratified turbulence;

- the study of the transfer of a moderately strong stratified turbulence to a weak relict turbulence, and investigation of the relative role of internal waves and turbulence proper at a very large time of evolution.

1. BASIC EQUATIONS, SCALES, AND PARAMETERS

The second-order model of the homogeneous turbulence of a steadily stratified Boussinesq liquid is comprised of the following differential equations (the details of the account for gravitation in various model equations are omitted).

1.1. Model equation for the tensor of Reynolds stresses

$$\frac{D}{Dt}\overline{u_iu_j} - P_{ij} + 2\epsilon^{ij} - \Phi_{ij} = -\beta(g_i\overline{u_jt} + g_j\overline{u_it}),$$

where

$$P_{ij} = -(\overline{u_iu_k}\,dU_j/dx_k + \overline{u_ju_k}dU_i/dx_k)$$

is the second-rank tensor of generating Reynolds stresses due to the averaged shear of the mean velocity;

$$\epsilon^{ij} \equiv \nu(-\Delta\overline{u_iu_j'})_{\xi=0} = \frac{1}{3}(da_{ij} + \delta_{ij})\epsilon_u$$

is the model relationship for the rate of dissipation of Reynolds stresses;

$$\epsilon_u = \epsilon^{ii}$$

is the dissipation of the turbulence kinetic energy;

$$a_{ij} = 3\overline{u_iu_j}/\overline{q^2} - \delta_{ij}$$

is the deviator of the tensor $\overline{u_iu_j}$;

$$\Phi_{ij} = \frac{1}{\rho}\overline{p\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)} = \Phi_{ij}^{(1)} + \Phi_{ij}^{(2)} + \Phi_{ij}^{(3)} =$$

$$= -a_u(1-d)a_{ij}\epsilon_u - b_u[\alpha_u\overline{q^2}S_{ij} + (\beta_u b_{ij} + \gamma_u c_{ij})P_{kk}] -$$

$$-\frac{1}{10}\beta g_k[2\overline{u_kt}\delta_{ij} - 3(\overline{u_it}\delta_{jk} + \overline{u_jt}\delta_{ik})]$$

is the model relation for the second-rank tensor of interaction between pressure fluctuations and gradients of velocity fluctuations, the three components of which determine, respectively, a "slow" return to isotropy, "rapid" deformation of turbulence by the mean shear and the change in the Reynolds stresses due to gravitation;

$$\overline{q^2} = \overline{u_i^2}$$

is the doubled total kinetic energy of turbulence;

$$S_{ij} = \frac{1}{2}(dU_i/dx_j + dU_j/dx_i)$$

is the second-rank tensor of the mean shear;

$$b_{ij} = 3P_{ij}/P_{kk} - \delta_{ij}$$

is the deviator of the tensor P_{ij};

$$c_{ij} = 3D_{ij}/P_{kk} - \delta_{ij}$$

is the deviator of the tensor D_{ij};

$$D_{ij} = -(\overline{u_iu_k}dU_k/dx_j + \overline{u_ju_k}dU_k/dx_i);$$

$$\beta = -(1/\bar{\rho}_0)(d\bar{\rho}/dT)_0$$

is the coefficient of the medium thermal expansion; a_u, b_u, d_u, α_u, β_u, γ_u are model coefficients, which, in general, are certain functions of the governing parameters; in the homogeneous turbulence considered, the turbulent Prandtl number

$$R_\lambda = \overline{q^2}^{1/2}\lambda_u/\nu,$$

is the parameter of the ratio the rate of turbulent kinetic energy generation to the rate of dissipation;

$$\overline{P_u} = P_{kk}/2\epsilon_u,$$

is the Taylor microscale of the length λ_u, which in the considered homogeneous anisotropic turbulence can be defined by the relation

$$\lambda_u^2 = 5\nu\overline{q^2}/\epsilon_u = 5\nu\tau_u,$$

where $\tau_u = \overline{q^2}/\epsilon_u$ is the time scale which is attributed to the turbulent velocity field.

1.2. The exact solution for the mean value of the squared scalar pulsations

$$\frac{D}{Dt}\overline{t^2} + 2(1-\overline{P}_t)\epsilon_t = 0,$$

where

$$\epsilon_t = \varkappa\,\overline{(\partial t/\partial x_k)^2} \equiv \varkappa\,(-\Delta_\xi\,\overline{tt'})_{\xi=0}$$

is the rate of "smearing" of scalar fluctuations;

$$\overline{P}_t = P_{tt}/2\epsilon_t$$

is the ratio of the rate of generation of scalar fluctuations due to the gradient vector of its mean value $P_{tt} = -2\overline{u_k t}\partial T/\partial x_k$ to the rate of destruction ϵ_t.

1.3. Model equation for the flow vector of a scalar value

$$\frac{D}{Dt}\overline{u_i t} - P_{it}\,I + \epsilon_{ii} - \Phi_{it} = -\alpha g_i\,\overline{t^2},$$

where

$$P_{it} = -(\overline{u_i u_k}dT/dx_k + \overline{u_k t}dU_i/dx_k)$$

is the vector of generation of the turbulent flux at the expense of the gradients of the mean values of velocity and scalar value;

$$\epsilon_{it} = d_u\frac{1}{\tau_{utw}}\overline{u_i t}$$

is the model relation for the rate of "smearing" of the flux $\overline{u_i t}$;

$$\Phi_{it} \equiv \frac{1}{\rho}\overline{p\frac{\partial t}{\partial x_i}} = \Phi_{it}^{(1)} + \Phi_{it}^{(2)} + \Phi_{it}^{(3)} =$$

$$= -a_{ut}(1-d_u)\frac{1}{\tau_{uts}}\overline{u_i t} +$$

$$+\frac{1}{5}(4dU_i/dx_k - dU_k/dx_i)\overline{u_i t} +$$

$$+\frac{1}{3}\alpha g_k(d_u a_{ik} + \delta_{ik})\overline{t^2}$$

is the model relation for the vector of interaction between the pressure fluctuations and the gradients of scalar fluctuations (the relations for $\Phi_{it}^{(2)}$ and $\Phi_{it}^{(3)}$ are the exact ones);

$$a_{ut}\tau_{uts}^{-1} = [4 - F_{us} + n_{ts}^{-1}(\overline{u_n^2}/\overline{q^2})R - \overline{P}_u]\tau_u^{-1}$$

is the model equation for the mixed time scale τ_u for $R_\lambda >> 1$, $P_\lambda >> 1$, $\overline{P}_u = \text{const}$, $\overline{P}_t = \text{const}$, $n_{ts}^{-1} \simeq 20$ is the velocity component in the direction of the scalar substance gradient;

$$\tau_{utw}^{-1} = [2(\sigma_{Taw0} + 3/5)R/R_{aw0} - \overline{P}_u]\tau_u^{-1}$$

is the model relation for the mixed time scale for $R_\lambda << 1$, $P_\lambda << 1$, $\overline{P}_u = \text{const}$, $\overline{P}_t = \text{const}$;

$$\sigma_{Taw0} = (3/10)\frac{1-\sigma}{\sigma}\bigg/\left[1 - \left(\frac{2\sigma}{1+\sigma}\right)^{3/2}\right]$$

is the asymptotic (for a large evolution time) value of the turbulent Prandtl number for $R_\lambda << 1$, $P_\lambda << 1$, $\overline{P}_t = \text{const}$, $\overline{P}_t \to \text{const}$ (the Deissler equation [8]); $P_\lambda = \overline{q^2}^{1/2}\lambda_t/\kappa$ is the turbulent Peclet number;

$$\lambda_t^2 = 6\kappa\overline{t^2}/\mathcal{E}_t = 6\kappa\tau_t$$

is the Taylor length microscale which refers to a scalar field;

$$\tau_t = \overline{t^2}/\mathcal{E}_t$$

is the time scale, attributed to a scalar field;

$$R_{aw0} = \frac{1}{5}\left[1 - 2\left(\frac{2\sigma}{1+\sigma}\right)^{3/2} + \sigma^{3/2}\right]\bigg/\left[1 - 2\left(\frac{2\sigma}{1+\sigma}\right)^{1/2} + \sigma^{1/2}\right]$$

is the parameter of the ratio of time scales τ_u/τ_t of the homogeneous turbulence for $R_\lambda << 1$, $P_\lambda << 1$, $\overline{P}_u = 0$, $\overline{P}_t = \text{const}$ (the exact solution of Dunn and Reid [9]); σ is the molecular Prandtl number.

1.4. Model equation for the rate of dissipation of the turbulent kinetic energy

$$\frac{D}{Dt}\mathcal{E}_u + (F_u^{**} - 3\overline{P}_u)\mathcal{E}_u^2/\overline{q^2} = -2d_u\beta g_i\frac{\sigma}{1+\sigma}\frac{1}{\tau_{utw}}\overline{u_i t},$$

where

$$(F_u^{**} - 3\overline{P}_u)\mathcal{E}_u^2/\overline{q^2} = \text{GEN} + \text{DIS} + \text{IN}$$

is the model relation for the sum of three effects: generation of $\mathcal{E}_u$ due to the mean shear (GEN = 0 for $R_\lambda >> 1$), destruction of the quantity $\mathcal{E}_u$ (or the vorticity $\overline{\omega_u^2} = 1/\nu\mathcal{E}_u$), variation $\overline{\omega_u^2}$ owing to the extension (compression) of vortex filaments (see the details in [7]).

The right-hand side of the equation involved simulates the effect of gravity on the vorticity of the field of velocity fluctuations.

The coefficient F_u^{**} associated with the combined effect of the vorticity destruction and the extension of the vortex filaments of an isotropic velocity field is simulated by a function of the turbulent Reynolds number:

$$F_u^{**}(R_\lambda) = F_{us}^{**}(1-d_u) + F_{uw}^{**} d_u,$$

where $F_{us}^{**} = 11/3$ and $F_{uw}^{**} = 14/5$ are for the asymptotic values for $R_\lambda >> 1$ and $R_\lambda << 1$, which are found analytically.

The parameter d_u is the empirical function of the turbulent Reynolds number; in the present work, it is assigned in the form

$$d_u = 1 - 2\left(1 + \sqrt{1 + \delta_u/R_\lambda^2}\right)^{-1}.$$

The remaining coefficients of the model presented are of the following form (see the detailed description in [7]):

$$b_u = 2\{1 + [(\alpha_s/F_{us}^{**} - 2)(1-d_u) + (\alpha_w/F_{uw}^{**} - 2) d_u]\overline{P}_u\}^{-1},$$

$$\alpha_u = 2\left(\frac{1}{11} d_u - \frac{1}{10}\right),$$

$$\beta_u = \frac{4}{33} d_u,$$

$$\gamma_u = -\frac{1}{33} d_u,$$

$$a_u \simeq 3,$$

$$\delta_u \simeq 2800,$$

$$\alpha_s \simeq 1,$$

$$\alpha_w \simeq 0{,}5.$$

1.5. Model equation for the rate of "smearing" of the scalar value fluctuations

$$\frac{D}{Dt}\epsilon_t = 2d\overline{P}_u\epsilon_t \frac{1}{\tau_u} + 2d\overline{P}_t \frac{1}{1+\sigma}\epsilon_t \frac{1}{\tau_{utw}} - (F_{t1}^* + F_{t2}^* R)\epsilon_t \frac{1}{\tau_u},$$

where the first two terms in the right-hand side of the equation simulate the effect of generation of the parameter ϵ_t due to the shear of the mean velocity and the gradient of the scalar mean value (in the case of strong turbulence this effect is absent); the second two terms simulate the combined effect of generation of the parameter ϵ_t due to the deformation of vortex filaments of the velocity field and "formations" of the scalar field associated with them and also the destruction of the parameter ϵ_t due to molecular diffusion.

The coefficients F_{t1}^* and F_{t2}^* are simulated by the following functions of the turbulent Reynolds and Peclet numbers and also the parameters R_λ and P_λ:

$$F_{t1}^* = F_{t1}^{**} - \overline{P}_u + d_u f_1 \overline{P}_u,$$

$$F_{t2}^* = F_{t2}^{**} - 2\overline{P}_t + d_u\left(\frac{2\overline{P}_t}{1+\sigma}\right) f_2,$$

$$f_1 = \left(1 - \frac{2\overline{P}_t}{1+\sigma}\right) + 1,$$

$$f_2 = \left[2\cdot\left(\sigma_{Taw0} + \frac{3}{5}\right) - \frac{4}{3}\left(R_{aw0} - \frac{3}{5}\right)\left(\frac{2\overline{P}_t}{1+\sigma}\right)^{-1}\right] R_{aw0}^{-1},$$

$$f_3 = \frac{4}{3}\left(R_{aw0} - \frac{3}{5}\right)\frac{R}{R_{aw0}}\left(1 - \frac{2\overline{P}_t}{1+\sigma}\right)\left(\frac{2\overline{P}_t}{1+\sigma}\right)^{-1}.$$

The coefficients F_{t1}^{**} and F_{t2}^{**} model the effects stated above in the case of isotropy of velocity and scalar value fields in the form of the following functions of the turbulent Reynolds number:

$$F_{t1}^{**} = F_u^{**} - 2 - \frac{4}{5} d_u,$$

$$F_{t2}^{**} = 2 + \frac{4}{3} d_u.$$

2. THE DYNAMICS OF THE TURBULENCE OF A STEADILY STRATIFIED LIQUID

In the present section, based on the second-order model considered above, the dynamics of the homogeneous turbulence of a steadily stratified liquid is investigated in the absence of the mean velocity shear. It is assumed that in an infinite liquid volume with a constant vertical density gradient $d\overline{\rho}/dx_2 = \mathrm{const}$ (the coordinate x_2 is in a vertical direction, opposite to the direction of the acceleration of gravity) the evolution of turbulence occurs from the initial state, corresponding to a three-dimensional homogeneous isotropic turbulence.

As is known, in the stratified liquid, when the state of equilibrium is violated, oscillations originate (internal gravitational waves) with the Brent-Väisälä frequency

$$N = \left(-\frac{g}{\overline{\rho}_0}\frac{d\overline{\rho}_0}{dx_2}\right)^{1/2}.$$

In the case of a turbulized liquid, oscillations of gravitational character involve liquid elements with a wide range of turbulent fluctuations, the turbulent modes with frequencies commensurable with the Brent-Väisälä frequency being found subjected to the effect of buoyancy forces. In the vertical direction, displacement of the turbulized element is estimated by the so-called scale of "build-up"

$$L_t = \overline{\rho^2}^{1/2}/(d\overline{\rho}_0/dx_2),$$

where $\overline{\rho^2}$ is the mean square density fluctuations. This length scale characterizes the maximum possible dimension of large turbulent modes in the stratified turbulence.

The "adjustment" of the turbulence developing in time to the internal gravitational waves can easily be illustrated on the example of past-grid turbulence. At small times of evolution, when the turbulence is still practically isotropic, the known relationships for the turbulent energy and Taylor length scales are maintained:

$$\overline{q^2} \sim \tau^{-1}, \quad \lambda_u \sim \tau^{1/2}.$$

At the same time it is easily seen that the turbulent Froude number (here, the value which is in inverse proportion to the generally accepted one is employed, so that in the absence of gravitation the value of the parameter defining the gravitation effect would be equal to zero)

$$\mathrm{Fr}_T = \mathrm{N}\,\lambda_u/\overline{q^2}^{1/2}$$

will evolve by the "law"

$$\mathrm{Fr}_T \sim \mathrm{N}t.$$

From this estimate it follows that at small evolution times, $\mathrm{Fr}_T \ll 1$, i.e., the gravitation does not affect the turbulence. For the time $t \sim N^{-1}$, the parameter Fr_T reaches a value of the order of unity, i.e., the inertial effects of turbulence and gravitation become commensurable. At these evolution times, wavelike variations of statistical parameters of turbulence must begin.

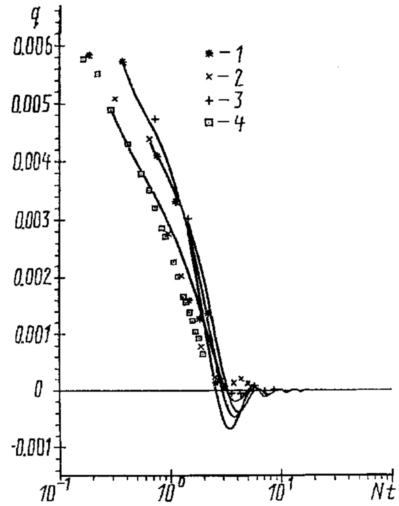


Fig. 1. Evolution of the transverse mass (or heat) flux $q = \overline{u_2 \rho}/UM(-d\bar{\rho}/dx_2)$: 1) $Fr = NM/U = 0.037$; 2) 0.063; 3) 0.152 (the experiment for water [4]); 4) $Fr = 0.024$ (the experiment for air [5]); solid lines indicate numerical modeling.

Experimental data obtained in recent years for the turbulence of a steadily stratified liquid [3-5], evolving in the wake behind the turbulizing grid, show that the wavy variation downstream of the time-averaged parameters is one of the peculiar features of steadily stratified turbulence evolution. As a result of the stated experiments, a series of conclusions have been made both about the phenomenon of passing over from the collapse to the assumed quasi-two-dimensional turbulence and inner gravitational waves at relatively small distances from the grid and also about the interaction between the turbulence and internal waves after the "collapse." However, the question of whether the three-dimensional turbulence under the gravity effect transfers into a quasi-two-dimensional or gravitational wave or into the "mixed" field of turbulence and internal waves still remains open, since this can be clarified only in the case of large evolution times which are as yet inaccessible either for direct modeling or for a physical experiment.

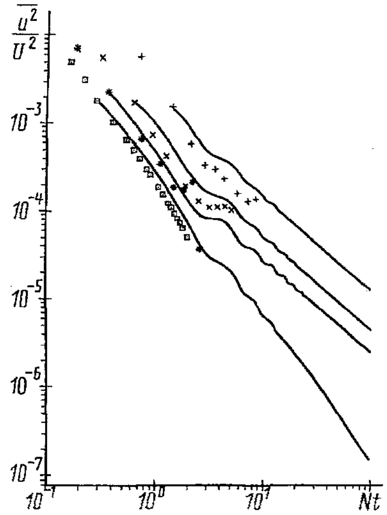
In the present work there is an attempt to apply an apparatus alternative to direct numerical modeling (the second-order model describing the dynamics of the coupled fields of velocity and density) for investigating the transition of three-dimensional turbulence in a steadily stratified medium to the final stage of evolution, in order to elucidate the field structure at this stage — whether this is a two-dimensional turbulence (as is predicted in certain theoretical works) or field of internal waves or a "mixture" of turbulent and wave modes.

Here, a fairly reasonable question arises: on what is the certainty based that the prediction of the behavior of the steadily stratified turbulence on the basis of the suggested model is adequate at large times of evolution? One can answer quite definitely that, first, it is based on the satisfactory agreement between predicted and experimental results in the case of strong turbulence of an unstratified liquid (see [10]), second, on the satisfactoriness of the model to an exact asymptotics [9] in the case of weak turbulence, i.e., at very large times of evolution, and, finally, on the triviality of the account for a gravitation effect model in differential equations.

Below, we shall try: to interpret the obtained numerical results, correlate them, when possible, with the experimental data of other authors; to investigate the degeneration of the steadily stratified turbulence after the "collapse," having elucidated here the role of the molecular Prandtl number; to consider the dynamic character of the steadily stratified turbulence at small values of the turbulent Reynolds and Peclet numbers, i.e., to learn, whether it really tends to a two-dimensional state, as follows from a number of theoretical research works.

2.1. The dynamics of a moderately strong turbulence of a steadily stratified liquid

Let us analyze the results of the numerical modeling of the moderately strong turbulence dynamics of a steadily stratified medium, realized in the experiments of Itsweire et al. [4] for water and of Lienhard and Van Atta [5] for air. Among a large number of experiments [4] carried out on water, let us choose three, corresponding to



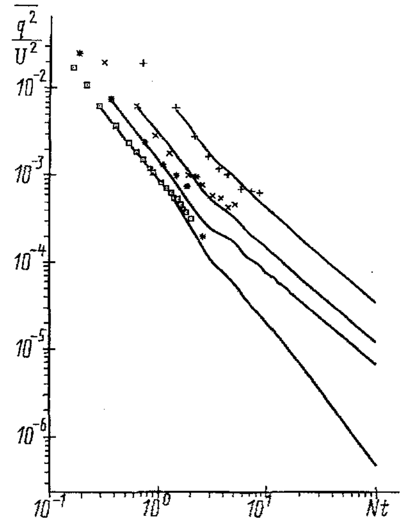


Fig. 2. Evolution of the kinetic energy of transverse velocity fluctuations $\overline{u_2^2}/U^2$; notation is that of Fig. 1.

Fig. 3. Evolution of the total doubled turbulent kinetic energy $\overline{q^2}/U^2$; notation is that of Fig. 1.

different values of the Froude number ($Fr = NM/U = 15.2 \cdot 10^{-2}$; $6.3 \cdot 10^{-2}$; $3.7 \cdot 10^{-2}$) with the dimension of the grid cell $M = 3.81 \cdot 10^{-2}$ m. Among the experiments conducted on the air [5], we take the experiment corresponding to the following parameters: $Fr = 2.4 \cdot 10^{-2}$, $M = 5.08 \cdot 10^{-2}$ m.

The numerical results presented below have been obtained as the solution to the Cauchy problem for a system of ordinary differential equations describing statistically mean values of the turbulence parameters: $\overline{u_2^2}$, $\overline{q^2}$, ϵ_u, $\overline{\rho^2}$, ϵ_ρ, $\overline{u_2\rho}$; for air, the last three parameters are replaced by the quantities $\overline{t^2}$, ϵ_t, and $\overline{u_2 t}$. The averaging sign here denotes the averaging over the ensemble of realizations. The initial conditions have been taken from the appropriate experiments.

The numerical results show that the turbulent flux of a scalar substance $\overline{u_2\rho}$ (Fig. 1) defining the source gravitational terms in the equations for the parameters $\overline{u_2^2}$, $\overline{q^2}$ and ϵ_u, performs fluctuations near the position $\overline{u_2\rho} = 0$; here, the first intersection of this position occurs for $Nt \simeq 2.5$; the period of fluctuations is estimated by the value $T \simeq 3.5N^{-1}$ independently of the values of the Froude number.

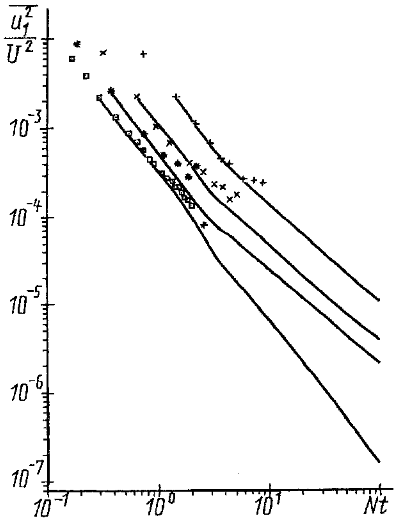
Fluctuations of the function $\overline{u_2\rho}$ directly cause fluctuations transverse fluctuations of the velocity $\overline{u_2^2}$ (see Fig. 2), and through this function - the turbulence kinetic energy (Fig. 3). Less noticeable are fluctuations of the mean value of the horizontal velocity fluctuations $\overline{u_1^2}$ (Fig. 4), since for them, the gravity effect reveals itself only by means of pressure fluctuations.

Let us note that the absence of the vertical turbulent transfer of mass (or heat) which begins when $Nt \simeq 30$ (see Fig. 1) by no means attests to the suppression of the vertical velocity fluctuations, which, as follows from Fig. 2, decay with a mean (over the amplitide of fluctuations) rate not exceeding the rate of degeneration of the velocity fluctuations of an unstratified liquid.

The fluctuations of the function $\overline{u_2\rho}$ directly affect also the behavior of the density fluctuations (Fig. 5), but not through a source gravitational term of nongradient type (which in this equation is, naturally, absent), but through the rate of generating density fluctuations due to the mean density value

$$P_{\rho\rho} = -2\overline{u_2\rho}d\bar{\rho}/dx_2.$$

For $Nt < 1$, the function $\overline{\rho^2}$ evolves similarly to the mean square of the passive scalar fluctuations (see, for example, [10]). When $Nt \simeq 2.5$, the decay of the parameter $\overline{\rho^2}$ begins qualitatively similarly to the decay of an isotropic passive scalar. The distinguishing feature from the isotropic case is the wavelike variation of density fluctuations with a period equal to the period of fluctuations of the transverse density flux.



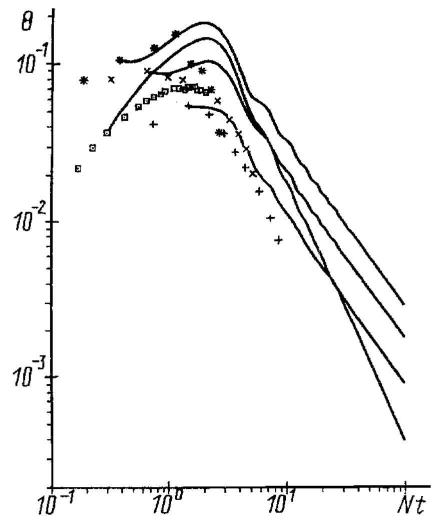


Fig. 4. Evolution of the kinetic energy of longitudinal velocity fluctuations $\overline{u_2^2}/U^2$; notation is that of Fig. 1.

Fig. 5. Evolution of the density (or temperature) fluctuations $\Theta = \overline{\rho^2}/M^2(d\bar{\rho}/dx_2)^2$; notation is that of Fig. 1.

As is known, the wave like variation of turbulence parameters in a medium steadily stratified over the density testifies to the presence of a field of internal waves induced by the gravitational forces. As is known, an indicator of the turbulence transition to internal gravitational waves is the value of the ratio of the kinetic energy of the vertical velocity fluctuations

$$K = \frac{1}{2}\bar{\rho}_0\overline{u_2^2} = \frac{1}{2}\bar{\rho}_0 N^2(\overline{u_2^2}/N^2)$$

to the potential energy of the density flux

$$P = \frac{1}{2}\bar{\rho}_0\frac{g}{\bar{\rho}_0}\frac{\overline{\rho^2}}{(d\bar{\rho}/dx_2)} = \frac{1}{2}\bar{\rho}_0 N^2\overline{\rho^2}/(d\bar{\rho}/dx_2)^2,$$

or the ratio of characteristic length scales

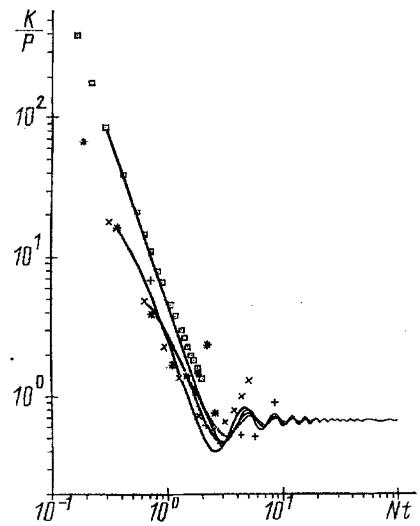
$$\frac{K}{P} = \left(\frac{L_b}{L_t}\right)^2,$$

where $L_b = (\overline{u_2^2}/N^2)^{1/2}$ is the buoyancy scale; $L_t = \overline{\rho^2}^{1/2}/(d\bar{\rho}/dx_2)$ is the "reversal" scale or characteristic vertical distance, at which the element of the turbulized liquid can be displaced from the state of equilibrium. The upper limit of the parameter L_t is the scale L_b, determined by the turbulence lag.

It is obvious that at small times of the strong turbulence evolution, the ratio L_b/L_t exceeds unity, the contribution of internal waves to the turbulent field being insignificant. The condition

$$\frac{K}{P} = 1$$

testifies to the "parity" of the turbulent disturbances of the velocity field in the vertical direction and internal waves; sometimes this condition is considered to be the condition of conversion of turbulenceto internal waves, assuming that finally (at a very large evolution time), the turbulence will completely pass over into internal waves; it will be shown below that this transition is not always possible.



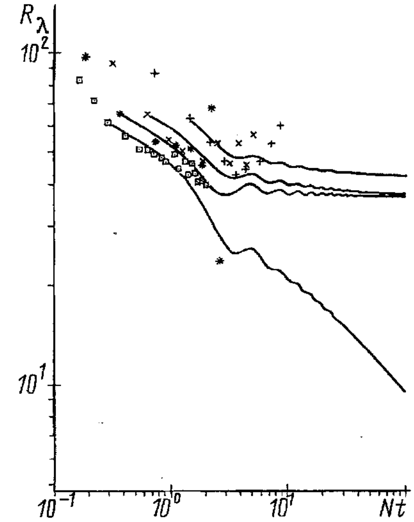


Fig. 6. Evolution of the ratio of the kinetic energy of vertical velocity fluctuations to the potential energy $K/P = (\overline{u_2^2}/N^2)/(\overline{\rho^2}/(d\bar{\rho}/dx_2)^2) = (L_b/L)_\rho^2$; notation is that of Fig. 1.

Fig. 7. Evolution of the turbulent Reynolds number $R_\lambda = \lambda_u \overline{q^2}^{1/2}/\nu$; notation is that of Fig. 1.

The curves presented in Fig. 6 show that, in the region of a relatively strong turbulence (the turbulent Reynolds and Peclet numbers are presented in Figs. 7 and 8), the ratio of the scales L_b/L_t, similarly to a transverse mass flux, is a fluctuating function reaching the asymptote. At small values of the dimensionless time, $L_b/L_t > 1$, which is indicative of the dominating contribution of turbulence to the disturbed field. The condition $L_b/L_t = 1$ corresponds to the coordinate $Nt \simeq 2$.

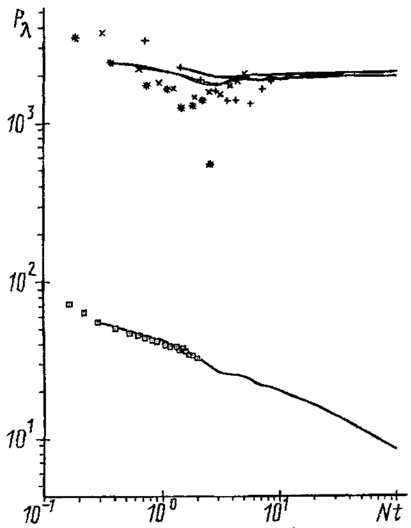
A minimum value of the function K/P corresponds to the coordinate $Nt \simeq 2.5$, i.e., to the point where the transverse mass flux q becomes zero for the first time. The asymptotic value reached by the ratio K/P equals approximately 0.7.

Since, as shown above, for $Nt \simeq 2$ the contribution of random internal waves to the superposition field becomes equally justified with the turbulence proper, starting with this point, one should expect the specific features of the dynamics of the considered parameters to be distinctive for a stratified liquid. Actually, from the data presented in Fig. 7, it is clear that for the value $Nt \simeq 2.5$, corresponding to the "collapse" of the transverse mass flux, a stepwise decrease in the rate of degeneration of the turbulence lag is observed. It is caused, evidently, by the internal waves with a large period (in proportion to the Brent-Väisälä period $T_{BV} = 2\pi/N$), which is shown by the stepwise change in the growth of the Taylor macroscale of the velocity field (Fig. 9) for $Nt \simeq 2.5$.

Since the internal random waves are less dissipative structures than the turbulence, then, beginning from the same coordinate ($Nt \simeq 2.5$), a slower degeneration of velocity fluctuations is observed (see Figs. 2-4).

2.2. The effect of the molecular Prandtl number

Realizations of the experiments and the appropriate modeling results considered in the present work refer to the moderately strong turbulence of a velocity field, which is confirmed by the plot of evolution of the turbulent Reynolds number (Fig. 7). At the same time, the turbulence of a scalar field is moderately strong for air and is fairly strong for water (see Fig. 8). In connection with this, one can expect that even at approximately equal values of the total Froude number, but different values of the molecular Prandtl number, the dynamics of turbulent parameters must differ. As follows from the figures considered above, in the near region ($Nt < 1$), where the effect of buoyancy forces on the velocity field is insignificant, the turbulent parameters behave similarly to the case of a passive scalar (see, for example, [10]): the velocity field parameters evolve in a self-similar way and, naturally, independently of



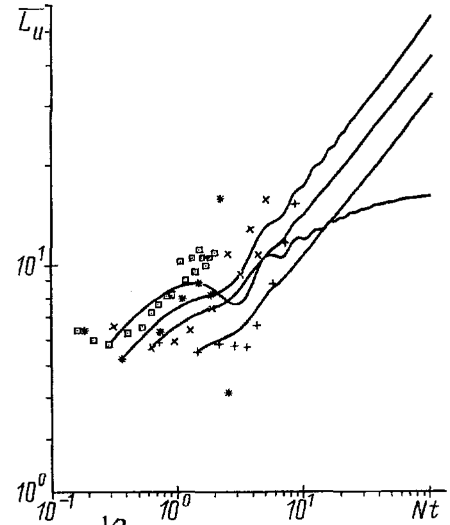


Fig. 8. Evolution of the turbulent Peclet number $P_\lambda = \lambda_\rho \overline{q^2}^{1/2}/\kappa$; notation is that of Fig. 1.

Fig. 9. Evolution of the Taylor macroscale of the velocity fluctuations $\overline{L}_u = L_u/M$; notation is that of Fig. 1.

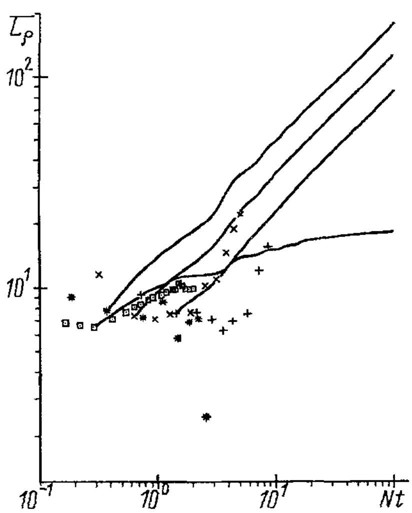


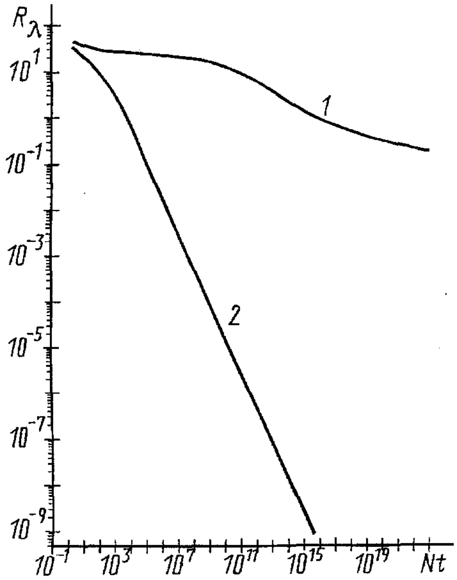
Fig. 10. Evolution of the Taylor macroscale of density fluctuations $\overline{L}_\rho = L_\rho/M$; notation is that of Fig. 1.

σ; the dynamics of the scalar field parameters, in particular, of the parameters Θ and L_t (Fig. 10), depends on the initial values of the ratio of the scales

$$R^0 = \frac{\tau_u^0}{\tau_t^0} = \frac{\overline{q^2}^0}{\overline{\rho^2}^0}\frac{\epsilon_\rho^0}{\epsilon_u^0} = \frac{6}{5}\frac{L_u^0}{L_\rho^0},$$

which were sufficiently different in the experiments for air and water. In other words, the difference between the power exponents in the laws of the evolution of turbulent parameters of a scalar field in the region $Nt < 1$ is due not to the difference in the values of σ, but to the initial value of the scale ratio R^0.

In the region of "collapse," as stated above, a "break" in the laws of evolution of turbulent parameters occurs caused by the appearance of internal waves. After that "break," evolution of the parameters of the field representing the "mixture" of the turbulence proper and random internal waves occurs conditionally in a self-similar way, i.e., in a self-similar way for the parameters leveled over the amplitude of fluctuations. However, the rate of self-similar



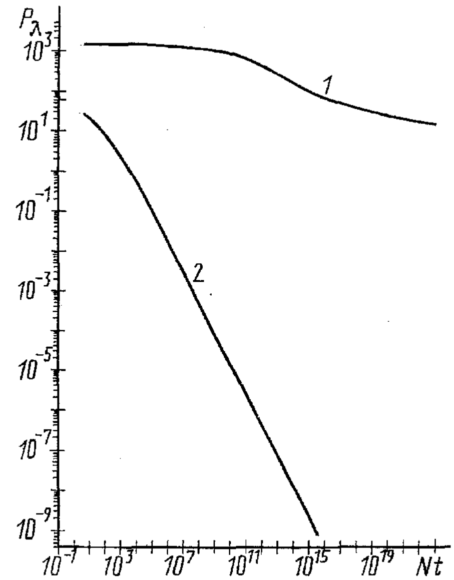


Fig. 11. Evolution of the turbulent Reynolds number at the transient and final stages: 1) the water; 2) the air.

Fig. 12. Evolution of the turbulent Peclet number at the transient and final stages: 1) the water; 2) the air.

evolution of the corresponding parameters differs considerably for air and water. Thus, from Fig. 3 it follows that the indicator of the rate of degeneration of the averaged (over the fluctuation amplitude) kinetic turbulent energy n in the power "law"

$$\overline{q^2} \sim (Nt)^{-n}$$

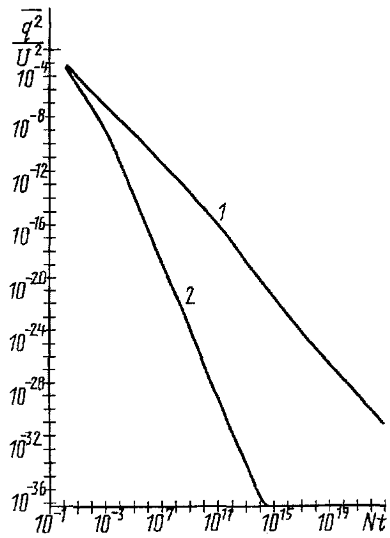
is approximately equal to 0.8 for water and n ≃ 1.63 for air. Such a sufficient difference in the rate of the kinetic energy degeneration after the "collapse" can be conditioned, on the one hand, by various contributions to the superposition field of random internal waves: a low rate of decay of the parameter $\overline{q^2}$ for water indicates the large contribution to this parameter of the energy of random internal gravitational waves, promoting the creation of large-scale weakly dissipating inhomogeneities.

On the other hand, the relatively high rate of degeneration of the kinetic turbulent energy in air, in comparison with water and also with the case of an unstratified medium, naturally, cannot be associated with the presence of internal waves. Apparently, it is explained by the relatively low lag of a scalar field (the evolution of the turbulent Peclet number is presented in Fig. 8). As is known, a weaker turbulence lag is accompanied by its more rapid degeneration. Therefore, in air, an earlier, than in the liquid, transition to the final stage can be observed.

Thus, the effect of the molecular Prandtl number on the evolution of the velocity field parameters is revealed through the lag of a scalar field, i.e., the turbulent Peclet number P_λ: the larger the value of this parameter, the greater is the contribution of internal waves to the disturbed velocity field.

From Fig. 5 it follows that the molecular Prandtl number also exerts an effect on the degeneration of the mean square of the scalar fluctuations; here, starting from the value of Nt corresponding to the "collapse," the rates of degeneration of the averaged values of $\overline{u_2^2}$ and $\overline{\rho^2}$ (or $\overline{t^2}$ for air) over the amplitude of fluctuations are found identical for the given value of σ, so that the ratio of the kinetic energy to the potential one, being a fluctuating function of time, has a constant mean value which depends neigther on the total Froude number, nor on the molecular Prandtl number (see Fig. 6).

It seems that this fact, which has not been reflected in the published works on this subject, is of great importance for generalizing the dynamics of a moderately strong homogeneous turbulence of a steadily stratified liquid.



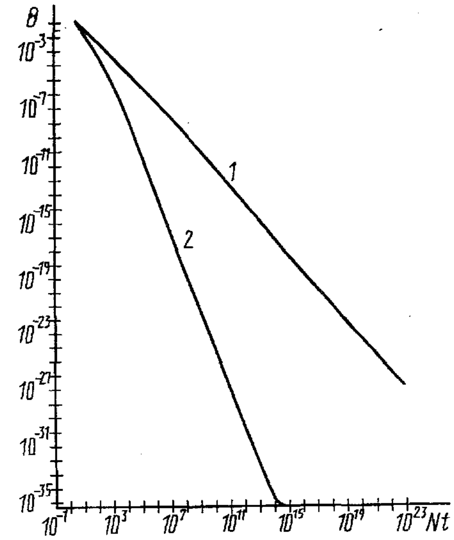


Fig. 13. Evolution of the total turbulent kinetic energy at the transient and final stages: 1) water; 2) air.

Fig. 14. Evolution of density fluctuations at the transient and final stages: 1) water; 2) air.

2.3. Transition of the turbulence of a steadily stratified liquid to a final stage of evolution

As is known from the dynamics of the homogeneous turbulence of a unstratified liquid, and also in compliance with the results of the present work, at large times of evolution the parameters R_λ and P_λ (see Figs. 7, 8) decay, i.e., in the process of evolution, the turbulence of a steadily stratified liquid loses its lag, when passing over to the final stage of degeneration, or, in correspondence with the oceanographical terminology, to the state of relict turbulence.

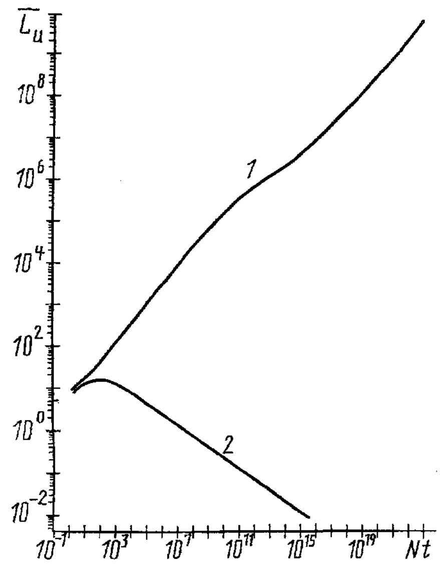
To understand the nature of relict turbulence, of fundamental importance is the study of turbulence evolution over the transition region, i.e., at moderate values of the parameters R_λ and P_λ.

For a detailed investigation of the evolution of a steadily stratified turbulence in this region of the parameters R_λ and P_λ, in the present work an analysis has been conducted of the numerical modeling results of two realizations of the experiment, which are distinguished by the values of the molecular Prandtl number and correspond to slightly varying values of the total Froude number: Itsweire et al. [4] for $\sigma \simeq 900$, $Fr = 3.7 \cdot 10^{-2}$ and Lienhard and Van Atta [5] for $\sigma \simeq 0.73$ $Fr = 2.4 \cdot 10^{-2}$. As follows from Figs. 11 and 12, approximation of the stratified turbulence to the final stage for the realizations considered occurs with a considerably different velocity: for a liquid, the state of a moderately strong turbulence is maintained up to $Nt \simeq 10^{11}$, whereas for air, the final stage of degeneration begins already at $Nt \simeq 10^4$.

In accordance with the different character of the increase in time of the lag of the velocity and scalar fields, the turbulent energy (Fig. 13) for the media considered decays unequally. At the final stage of degeneration $\overline{q^2}$ for air, which is established when $Nt \simeq 10^4$, the indicator of the degeneration rate in the power law equals, just as for an unstratified medium, $n = -5/2$. For a liquid, the degeneration rate indicator $\overline{q^2}$ at the final stage, established when $Nt \simeq 10^{17}$, has a value approximately one half of that for air.

The decay occurs qualitatively similarly to the turbulent kinetic energy in the far region of density (or temperature) fluctuations, presented in Fig. 14, in the domain of small values of the parameters R_λ and P_λ, the values of the indicators of degeneration degree of the parameter $\overline{\rho^2}$ for water and $\overline{t^2}$ for air being here approximately equal to the appropriate values of the indicator of degeneration of the parameter $\overline{q^2}$.

To understand the character of the velocity and scalar fields in the far region, of fundamental importance is the behavior of length scales of power-containing vortices, i.e., the Taylor macroscales L_u and L_ρ. As follows from Figs. 15 and 16, the behavior of the macroscales for air and water is in principle different: whereas for air the parameters L_u and L_t evolve similarly to the isotropic case (or to the case of the uniform field of a passive scalar for



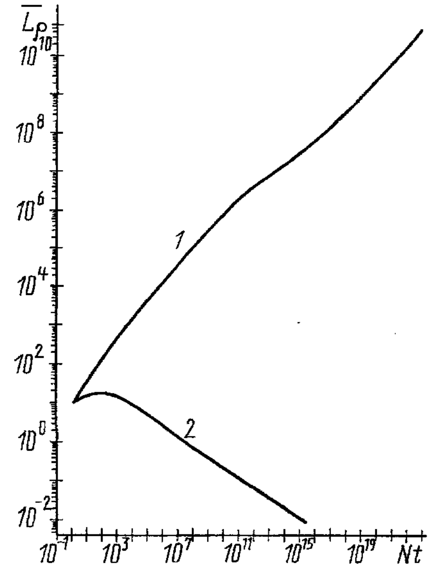
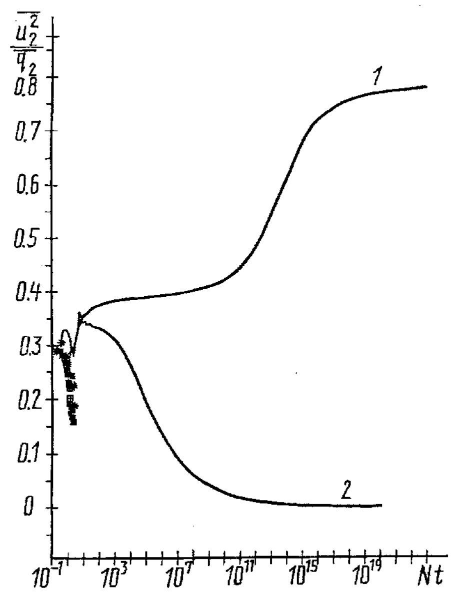


Fig. 15. Evolution of the Taylor macroscale of velocity fluctuations at the transient and final stages: 1) water; 2) air.

Fig. 16. Evolution of the Taylor macroscale of density fluctuations at the transient and final stages: 1) water; 2) air.



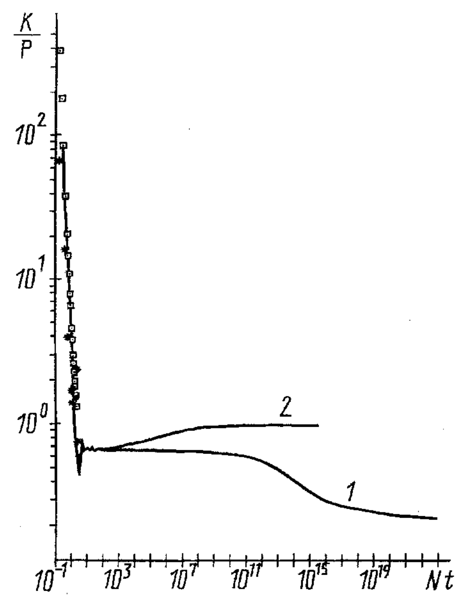


Fig. 17. Evolution of the ratio of the energy of transverse velocity fluctuations to the total kinetic energy at the transient and final stages; notation is that of Fig. 1: 1) water; 2) air.

Fig. 18. Evolution of the ratio of the kinetic energy of transverse velocity fluctuations to the potential energy at the transient and final stages; notation is that of Fig. 1: 1) water; 2) air.

the parameter L_t), for water the indicated parameters grow infinitely. This fact can presumably be explained by the different contribution of internal waves to the far region for the two cases involved: for Nt >> 1, the effect of internal waves on air turbulence is insignificant, whereas in the liquid, internal waves become dominating. Although such an assumption is of a heuristic character, it is confirmed by the analysis of the evolution of other turbulent parameters. One of the most impressive pictures is the evolution of the parameter of the ratio of the kinetic energy of vertical fluctuations to the total kinetic energy of the velocity field (Fig. 17). As follows from the numerical results presented here, when Nt >> 1, the vertical velocity fluctuations in air appear earlier than in the case of longitudinal and lateral

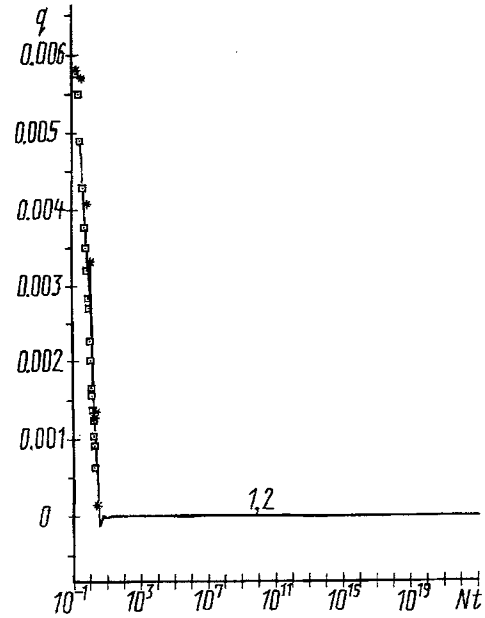


Fig. 19. Evolution of the transverse mass (or heat) flux q at the transient and final stages; notation is that of Fig. 1.

fluctuations, i.e., the velocity field becomes two-dimensional. For water, the picture is the opposite one: at large values of Nt, vertical velocity fluctuations decay more slowly than the other two components, so that the parameter $\overline{u_2^2}/\overline{q^2}$ approaches unity, i.e., the velocity field in the case considered asymptotically approximates a quasi-one-dimensional one.

The presumed picture of the structure of the relict turbulence described above is confirmed by the analysis of the evolution of the ratio of the kinetic energy of vertical velocity disturbances to the potential energy (Fig. 18). It follows from the figure that in the case of turbulence formation in a liquid, the parameter K/P asymptotically (for $Nt \to \infty$) tends to zero (or a certain small value), which testifies to the dominating role of internal waves in the superposition velocity field for $Nt \gg 1$. In the case of the air medium, for $Nt \gg 1$ the parameter K/P where the contribution of internal waves to the superposition velocity field somewhat exceeds the contribution of the turbulence proper, asymptotically approaches the value $K/P = 1$ from the region $K/P < 1$, indicating a "parity" of the energy of vertical turbulent disturbances and internal waves. However, as is seen from Fig. 17, the contribution of vertical disturbances (both of the turbulence and of internal waves) to the three-dimensional velocity field is insignificant (the turbulence tends to a two-dimensional or horizontal one), i.e., in Fig. 18, only the curve corresponding to the liquid is of informative character as regards the analysis of the contribution of internal waves to the relict turbulence: it indicates the dominating role of internal waves in the relict turbulence of a steadily stratified viscous liquid.

Thus, the analysis of the numerical modeling results for the evolution of the turbulence in steadily stratified media shows that depending on the molecular Prandtl number the relict turbulence, i.e., the turbulence of steadily stratified media for $R_\lambda \ll 1$ and $P_\lambda < 1$, can represent either a quasi-two-dimensional field with a predominant contribution of random two-dimensional disturbances and an insignificant "admixture" of internal waves (for gaseous media) or a quasi-one-dimensional field with a predominant contribution of quasi-one-dimensional (vertical) internal waves and an insignificant "admixture" of three-dimensional turbulence (for liquid media).

3. THE STRUCTURE OF THE TURBULENCE OF STEADILY STRATIFIED MEDIA

The analysis presented above allows one to assume that the disturbed field of steadily stratified media represents the "mixture" of the turbulence proper and internal gravitational waves, and depending on the molecular Prandtl number, the contribution of internal waves to the superposition field is diverse. It should be noted, however, that the conclusion made above about the contribution of internal waves into the disturbed velocity and scalar fields for $Nt \gg 1$ is of heuristic character, since the functions considered above represent in the far region smooth curves

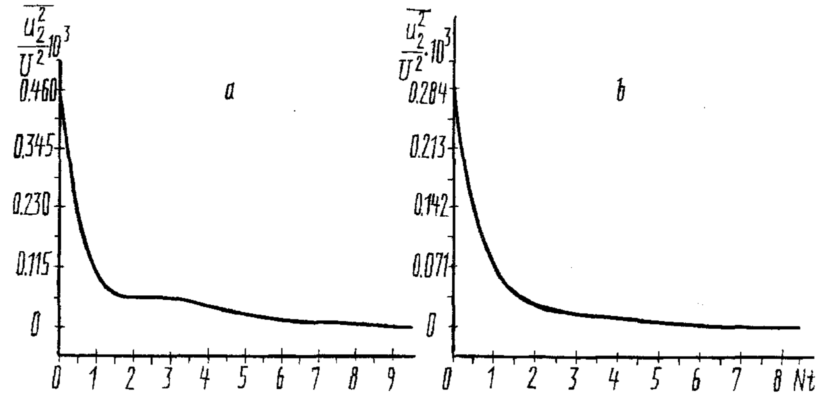


Fig. 20. Evolution in the near region of the energy of vertical velocity fluctuations: a) water, $(Nt)_0 = 1.0$, $(\overline{u_2^2}/U^2)_{min} = 2.53 \cdot 10^{-5}$; b) air, $(Nt)_0 = 1.0$, $(\overline{u_2^2}/U^2)_{min} = 7.56 \cdot 10^{-6}$.

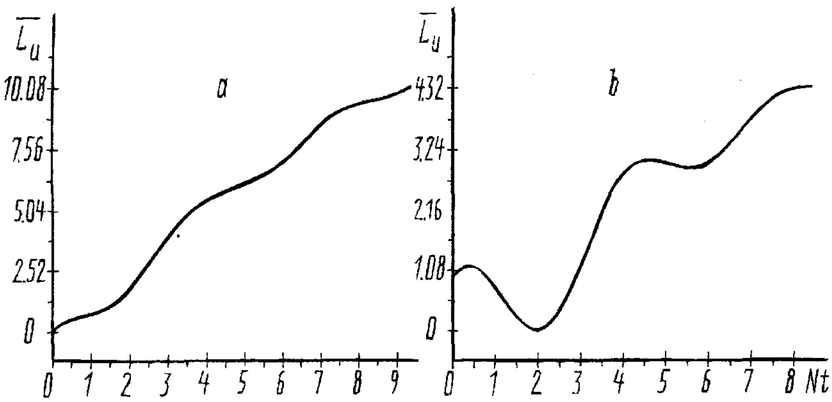


Fig. 21. Evolution in the near region of the Taylor macroscale of the velocity field: a) water, $(Nt)_0 = 1.0$, $(L_u/M)_{min} = 7.41$; b) air, $(L_u/M)_{min} = 7.6$.

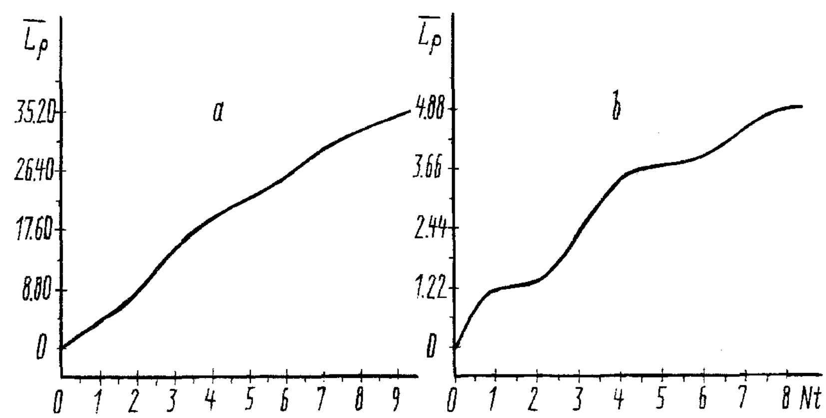


Fig. 22. Evolution in the near region of the Taylor macroscale of the density field: a) water, $(Nt)_0 = 1.0$; $(L_\rho/M)_{min} = 13.9$; b) air, $(L_\rho/M)_{min} = 10.06$.

not containing signs of wavelike motion either in a liquid or in a gas (see Figs. 17-19). Moreover, the evolution of a transverse mass flux, which is an indicator of the presence of wavelike motion of the medium, demonstrates (see Fig. 19) the absence in the far region of any signs of anisotropy of a scalar field at all.

In this connection, the necessity arises to analyze in detail in the process of time evolution the character of change of the parameters of a disturbed field with short periods of the argument Nt, which could give a verification of the existence of internal waves and would make it possible to elucidate their contribution to the disturbed fields of velocity and density.

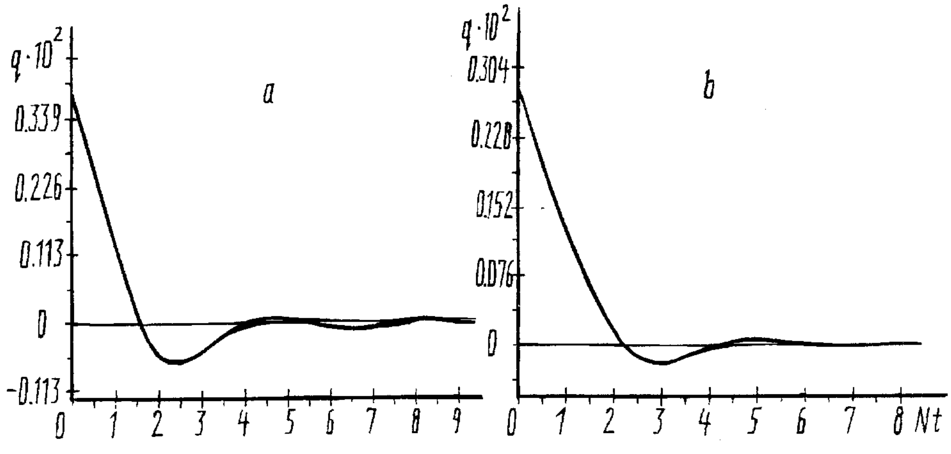


Fig. 23. Evolution in the near region of the transverse turbulent density flux $q=\overline{u_2\rho}/(UMd\bar{\rho}/dx_2)$: a) water, $(Nt)_0=1.0$, $q_{min}=-0.715\cdot10^{-3}$; b) air, $(Nt)_0=1.0$, $q_{min}=-0.2\cdot10^{-3}$.

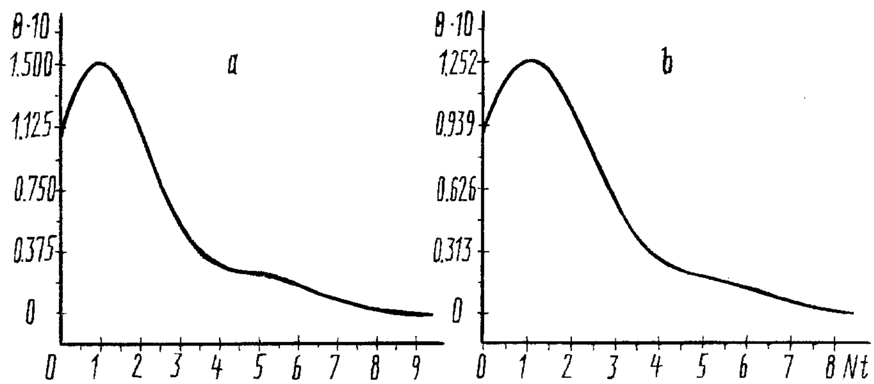


Fig. 24. Evolution in the near region of density fluctuations $\Theta=\overline{\rho^2}/M^2(d\bar{\rho}/dx_2)^2$: a) water, $(Nt)_0=1.0$, $\Theta_{min}=3.1\cdot10^{-2}$; b) air, $(Nt)_0=1.0$, $\Theta_{min}=1.88\cdot10^{-2}$.

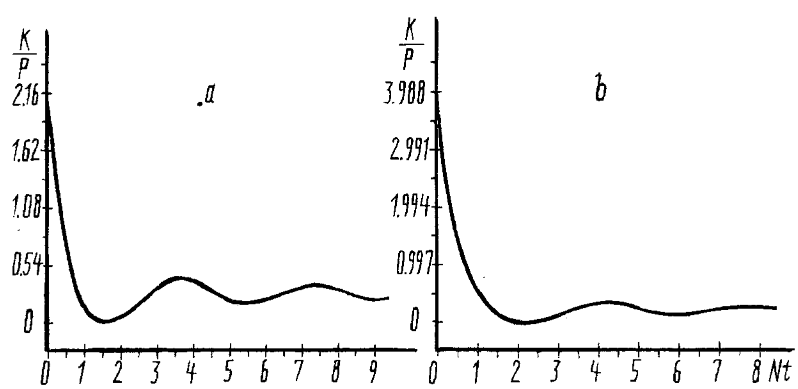


Fig. 25. Evolution in the near region of the ratio of the kinetic energy of the vertical velocity fluctuations to the potential energy $K/P = (L_b/L_\rho)^2 = (\overline{u_2^2}/N^2)/\overline{\rho^2}/(d\bar{\rho}/dx_2)^2$: a) water, $(Nt)_0=1.0$, $K/P=0.394$; b) air, $(Nt)_0=$

In accordance with the data considered in previous sections, for $Nt < 1$ the turbulent parameters of a steadily stratified medium evolve qualitatively similarly to the case of independent velocity and scalar fields. When $Nt \simeq 2.5$-3.0, a conventional collapse of turbulence occurs, determined by vanishing of the transverse mass flux and indicating the formation of internal gravitational waves.

In view of the coherence of the velocity and density fields in a stratified medium, the presence of internal waves in the "mixed" field must be revealed in the evolution of all turbulent parameters referring both to the velocity field and to the density field. Actually, as follows from Figs. 20-25, already at the early stage of evolution of the

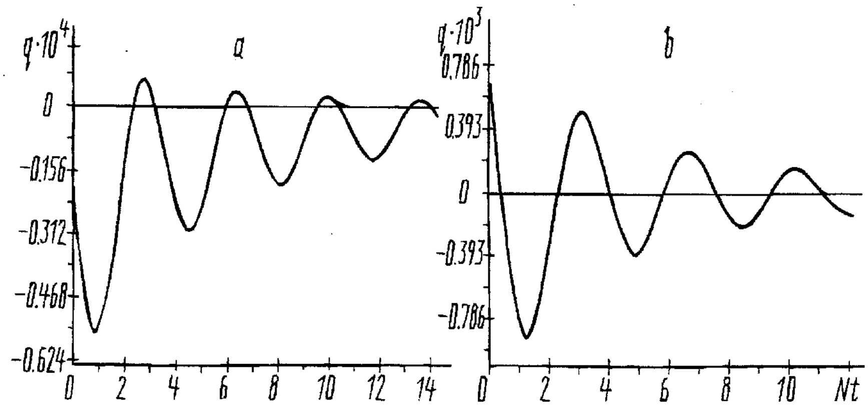


Fig. 26. Evolution of the transverse mass (or heat) flux $q=\overline{u_2 \rho}/UM(-d\bar{\rho}/dx_2)$ at the beginning of the transient stage: a) water, $(Nt)_0 = 10.0$; b) air, $(Nt)_0 = 10.0$.

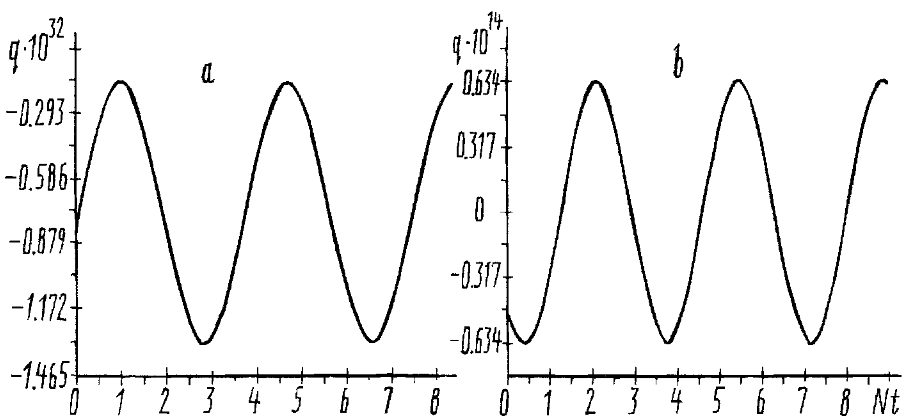


Fig. 27. Evolution at the transient stage of the transverse turbulent mass flux: a) water, $(Nt)_0 = 10^{14}$; b) air, $(Nt)_0 = 10^5$.

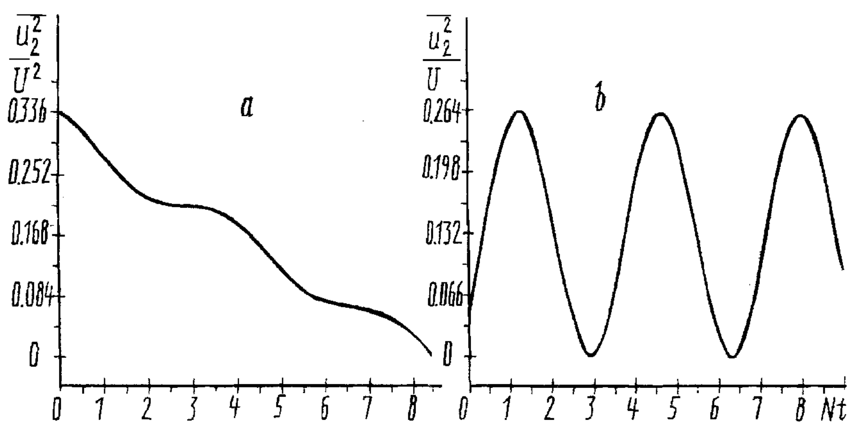


Fig. 28. Evolution at the transient stage of the energy of transverse velocity fluctuations: a) water, $(Nt)_0 = 10^{14}$; b) air, $(Nt)_0 = 10^5$.

steadily stratified turbulence comprising also the region of Nt to the "collapse," wavelike change of the turbulent parameters occurs, including also fluctuations of the function q relative to the zero value. Comparison of Figs. 25a and 25b shows that at the initial stage of evolution the contribution of internal waves to the disturbed field for a gas is somewhat smaller than for a liquid. The period of fluctuations of functions at the initial stage of evolution does not depend on the molecular Prandtl number and is approximately equal to

$$T \simeq 0{,}56 T_{BV}.$$

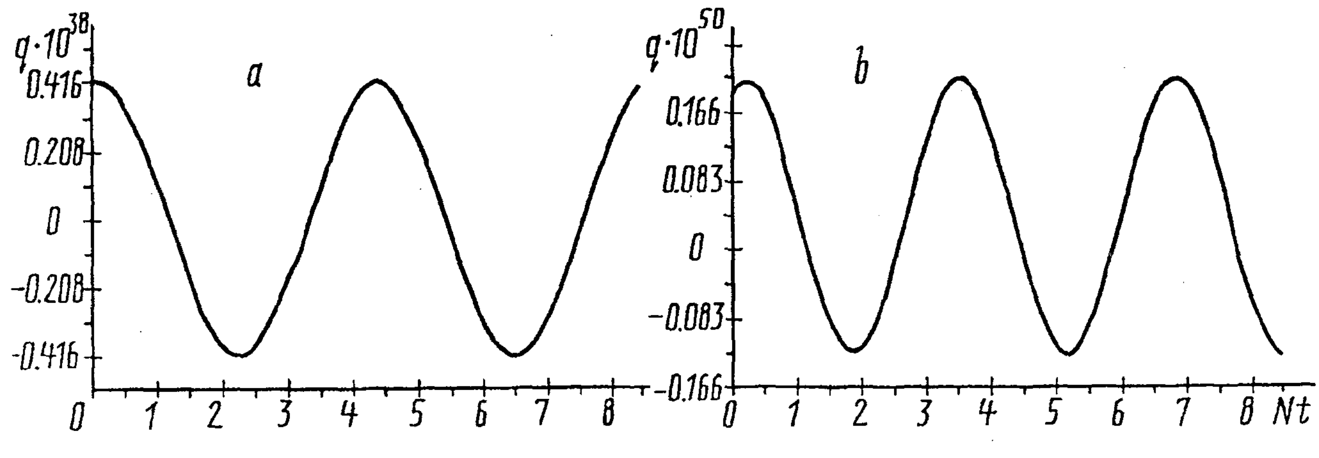


Fig. 29. Evolution at the final stage of the transverse density (or heat) flux: a) water, $(Nt)_0 = 10^{22}$; b) air, $(Nt)_0 = 10^{14}$.

A peculiar feature of turbulence development in a steadily stratified liquid in the initial region is a counter-gradient vertical mass flux. Actually, as follows from Fig. 26a, the value of the parameter q averaged over a certain number of "periods" is negative. In the same region of the dimensionless time Nt the considered parameter for a gas is equal on the average to zero.

In the transient region, determined by moderate values of the turbulent Reynolds and Peclet numbers and existing in the range 10^{10}-10^{20} for a liquid and 10^2-10^{12} for air (see Fig. 18), the tendency in the development of the parameter q (see Fig. 27) is maintained, i.e., one can observe a counter-gradient transverse mass flux for a liquid and (on the average) zero heat flux for air. A wavelike behavior is characteristic also for other turbulent parameters in the transient region (Fig. 28), the period of fluctuations being invariable throughout the whole range of Nt considered above, including the initial region of evolution.

At the final stage of evolution, defined by small values of the turbulent Reynolds and Peclet numbers and existing in the range $Nt > 10^{20}$ for a liquid and $Nt > 10^{12}$ for air, a wavelike character of evolution of the transverse turbulent flux of mass or heat is maintained (see Fig. 29). However, the qualitative difference between the evolution of this function at the stage considered and the previous one consists in the absence (on the average) of the turbulent mass flux for a liquid and in the re-establishment of the "gradient" heat flux for air. Here the period of fluctuations of the function q(Nt) for the water somewhat increased:

$$T \simeq 0{,}7T_{\mathrm{BV}};$$

for air, the period of fluctuation practically did not change.

Thus, both in the transient region and at the final stage of the evolution of the temperature-stratified turbulence the gravity plays the main role in the formation of the disturbed velocity field:

at large values of the molecular Prandtl numbers characteristic for liquid media, internal waves play the predominant role in the vertical disturbances of the velocity field; decaying slower than the turbulent velocity fluctuations due to the lower dissipation of large-scale structures of wavelike character, internal waves lead to the formation for $Nt \to \infty$ of a quasi-one-dimensional field, in which the intensity of vertical disturbances conditioned by the internal waves exceeds the intensity of longitudinal fluctuations by approximately an order;

at values of the molecular Prandtl number of the order of unity characteristic for gases, internal waves do not play a governing role in the superposition velocity field; the role of gravitation is reduced to the suppression of vertical velocity disturbances (both internal waves and turbulent fluctuations), so that when $Nt \to \infty$, the velocity field represents a quasi-two-dimensional (horizontal) turbulence with an insignificant "admixture" of internal waves.

CONCLUSIONS

The results are presented of the modeling of evolution of the homogeneous turbulence of a steadily stratified liquid at any distance from a certain arbitrary origin, say, from a turbulizing grid, which is the generator of a turbulent isotropic velocity field. The source of the turbulence of a scalar field is a constant transverse gradient of the mean density (or temperature), generating the disturbances of a scalar value. The cross-correlation of transverse (in the direction of the mean density gradient) velocity fluctuations and density fluctuations or the turbulent transverse flux of mass (or heat), being the source term in the equations for normal turbulent stresses and the rate of dissipation of the kinetic turbulent energy, determines, in essence, peculiar features of the evolution of the turbulent steadily stratified liquid.

Specific features of turbulence evolution in the initial region

1. The fundamental peculiar feature of the evolution of the considered homogeneous turbulence is a wavelike alternating-sign variation of the cross-correlation $\overline{u_2 \rho}$, i.e., interchange in time of "natural" and "counter-gradient" mass (or heat) fluxes with the period $T \simeq 3.5N^{-1} \simeq 0.5T_{BV}$, independent of the Froude number. The results of modeling performed in the present work show that this singular feature is distinguishing for flows with any (including arbitrarily small) values of Brent-Väisälä frequency. However, the amplitude of fluctuations of the indicated function decreases with decrease in the Froude number $Fr = NM/U$. Thus, concerning experimental verification of this peculiarity, one should bear in mind that at small values of the Froude number the amplitude of fluctuations of the parameter $\overline{u_2 \rho}$ can be sufficiently small, existing within the confidence interval of measurement, which makes it impossible to discern the presence of fluctuations directly from the experiment. Obviously, this situation is characteristic for definite, corresponding to small values of Fr, realizations of the experiments of Lienhard and Van Atta [5] and also Yoon and Warhaft [11], in which the sign alternation of the parameter $\overline{u_2 \rho}$ is not evident.

2. The sign of the transverse mass flux averaged over a large interval of the dimensionless time Nt depends on the contribution of internal waves to the disturbed field: with an insignificant role of internal waves, an ordinary turbulent transfer prevails; in the case of a significant contribution of internal waves, a counter-gradient transfer prevails (the sign of the turbulent mass flux is identical to the sign of the mean density gradient).

3. In the region under consideration, owing to the fluctuating character of the transverse mass flux, all statistical mean turbulent characteristics of both the velocity field and the scalar field evolve in a wavelike way with the same period as the function $\overline{u_2 \rho}$. Here, the value $\overline{u_2 \rho} = 0$, recurring with the period stated above, by no means implies the suppression of vertical disturbances of the velocity. Moreover, the significant contribution of internal waves, the evidence of which is a wavelike variation of turbulence parameters, leads to a slower degeneration of the energy of velocity fluctuations than in the case of a passive scalar. This is natural enough, since the superposition field containing internal waves is less dissipative (more large-scale) than a purely turbulent velocity field.

4. A significant contribution of internal waves to the "mixed" field is confirmed by a stepwise variation of the increase in Taylor macroscales of the velocity and scalar fields, starting with $Nt \simeq 2.5$, i.e., from the moment when the transverse mass flux becomes sign-alternating.

5. Values of the molecular Prandtl number distinctive for different media (air and liquid) determine a significant difference between the values of turbulent Peclet numbers. This explains the different rate of evolution of turbulent parameters in the experiments described above.

6. The value of the ratio of the kinetic energy of transverse velocity fluctuations to the potential energy of the field density, averaged over the amplitude of fluctuations, is a universal constant in the case of strong turbulence, depending neither on the Froude number nor on the molecular Prandtl number and approximately equal to 0.65.

Peculiar features of turbulence evolution when passing over to the final stage

1. The transient region is characterized by a significant non-self-similarity of evolution of turbulent parameters.

2. The dimension of the transient region depends, to a great extent, on the molecular Prandtl number: it is larger, the higher the value of σ. One cannot attribute this fact only to a relatively large turbulence lag in a liquid; the governing role, here, belongs to internal waves, the contribution of which to the superposition disturbed field is more significant in liquids than in gases.

3. In the region considered, the turbulent macroscales of velocity and density fields evolve with a fundamental difference for the two considered media: for air, these are functions with a maximum in the domain of moderate values of the parameters R_λ and P_λ, just as in the case of a passive scalar; for a liquid, these are functions infinitely growing over Nt. This fact is an indirect confirmation of a different contribution of internal waves to the superposition disturbed field in the far region for liquids and gases: in a steadily stratified gas, the contribution of internal waves to the disturbed field is insignificant, whereas the long internal waves in the disturbed far field of a steadily stratified liquid are predominant.

4. The above said is confirmed by a number of facts, which, in particular, include the value of the rate of degeneration of the turbulence energy in a gas (Fig. 13), the indicator of which in the power law is $n = -5/2$. For a liquid, it is approximately one-half, which testifies to the dominating role of larger, i.e., less dissipative, structures in the disturbed field formed in a steadily stratified liquid.

5. An important specific feature in principle of the turbulence evolution in various steadily stratified media is revealed in the ratio of the kinetic energy of transverse fluctuations to the total kinetic energy of velocity fluctuations (see Fig. 17): for a liquid, the value of this parameter tends to a value close to unity; for air - to zero. This means that the relict turbulence in a liquid is comprised, in general, of internal waves, and in a liquid - two-dimensional (horizontal) turbulence.

6. Evidence of the dominating contribution of internal waves to the superposition velocity field in a liquid in the case of a large time of evolution is the tendency to a certain value (less than unity) of the ratio of the kinetic energy of transverse velocity fluctuations to the potential one.

7. A qualitative difference between the disturbed velocity field in a liquid and in a gas at very large evolution times lies in the fact that the relict turbulence (i.e., the turbulence of steadily stratified media when $R_\lambda \ll 1$ and $P_\lambda \ll 1$) represents either quasi-two-dimensional random velocity disturbances (for gaseous media), or a quasi-one-dimensional velocity field induced by the predominant contribution of vertical internal waves for $\sigma \gg 1$.

The turbulent structure of steadily stratified media

1. The fluctuating character of the variation in time of turbulent parameters is a specific feature of the turbulence evolution in a steadily stratified medium as early as the early stage of its development, up to the moment of time when the parameter $\overline{u_2 \rho}$ vanishes for the first time.

2. At the beginning of turbulence development, the sign of the transverse mass flux fluctuations averaged over certain number of periods depends on the molecular Prandtl number: in a liquid, the counter-gradient mass transfer prevails, whereas in a gas the transverse mass flux is practically absent.

3. The fluctuation period of functions at the initial stage of evolution is independent of the molecular Prandtl number and approximately equals $T \simeq 0.56T_{BV}$.

4. In the transient region the character of variation in time of turbulent parameters is qualitatively similar to the turbulence evolution in the initial region.

5. At the final stage of turbulence evolution, the wavelike character of turbulent parameters of the velocity and scalar fields is maintained, but the amplitude of disturbances of parameters for a liquid considerably exceeds the amplitude of disturbances of the corresponding parameters for the air.

6. For liquid media the period of disturbances of functions at the final stage somewhat increases in comparison with the previous transient region.

7. The turbulent mass flux, averaged over the series of disturbance periods, for liquid media is practically absent; for a gas it acquires the usual sign associated with the gradient transfer.

The role of gravitation in the homogeneous turbulence of a steadily stratified liquid is revealed through the "participation" of internal waves in the "mixed" disturbed velocity field, which is reduced to the formation at large evolution times of a quasi-one-dimensional disturbed field with the predominance of wavelike disturbances in a liquid and a quasi-two-dimensional (horizontal) field with the prevalence of purely turbulent stochastic disturbances in a gas.

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NOTATION

D_{ij}, the second-rank tensor; $Fr = NM/U$, Froude number; F_u, the function of interaction; $L_t=\overline{\rho^2}^{1/2}/(d\overline{\rho}_0/dx_2)$, the "build-up" scale; $L_b=\overline{u_2^2}/N^2$, buoyancy scale; $K = (1/2)\,\overline{\rho}_0\overline{u_2^2}$, the kinetic energy of vertical fluctuations; M, the dimension of the grid cell; $N = [(-g/\overline{\rho}_0)/d\overline{\rho}/dx_2]^{1/2}$, Brent-Väisälä frequency; $P = (1/2)\overline{\rho}_0(g/\overline{\rho}_0)\overline{\rho}_2/(d\rho/dx_2)^2$, turbulent potential energy; $P_\lambda=\overline{q^2}^{1/2}\lambda_\rho/\kappa$, the turbulent Peclet number; P_{ij}, second-rank tensor of generating Reynolds stresses; $R_\lambda=\overline{q^2}^{1/2}\lambda_u/\nu$, the turbulent Reynolds number; $R = \tau_u/\tau_\rho$, the ratio of time scales; S_{ij}, second-rank tensor of the mean shear; $T_{BV} = 2\pi/N$, the Brent-Väisälä period; U_i, velocity vector; d_u, the parameter of the turbulent Reynolds number; a_u, absolute constant; a_{ut}, b_u, n_{ts}, variable coefficients; b_{ij}, deviator of the tensor P_{ij}; c_{ij}, deviator of the tensor D_{ij}; g_i, vector of the gravity force acceleration; p, t, pressure and temperature fluctuations; α_u, β_u, γ_u, variable coefficients; δ_u, the coefficient in the parameter d_u; Δ, Laplace operator; ϵ_u, rate of generation of velocity fluctuations; ϵ_t, rate of generation of temperature fluctuations; λ_t, the Taylor microscale of a scalar field; λ_u, the Taylor microscale of a velocity field; ν, kinematic viscosity; κ, molecular diffusion; $\sigma = \nu/\kappa$, the molecular Prandtl number; ρ, density fluctuations of a liquid; $\overline{\rho}$, mean density value; τ, time; $\tau_u = q^2/\epsilon_u$, time scale of the velocity field; $\tau_\rho=\overline{\rho^2}/\epsilon_\rho$, time scale of the density field; ξ, the vector of the distance between two points. Subscripts: s, the condition of strong turbulence; w, the condition of weak turbulence; T, belonging of a parameter to a turbulized medium; 0, the condition of absence of shear; a, asymptotic value; u, belonging of a function to the velocity field; t, belonging of a function to the temperature field; ρ, belonging of the function to the density field; Superscripts: $^-$, operator of ensemble averaging; $'$, belonging of a function to the second of the considered points.

REFERENCES

1. T. D. Dickey and G. L. Mellor, J. Fluid Mech., **89**, Pt. 1, 13-31 (1980).
2. J. J. Riley, R. W. Metcalfe, and M. A. Weissman, AIP Conf. Proc. 76 (ed. B. J. West) (1981), pp. 79-112.
3. D. C. Stillinger, K. N. Helland, and C. W. Van Atta, J. Fluid Mech., **131**, 91-122 (1983).
4. E. C. Itsweire, K. N. Helland, and C. W. Van Atta, J. Fluid Mech., **162**, 299-338 (1986).
5. J. H. Leinhard and C. W. Van Atta, J. Fluid Mech., **210**, 57-112 (1990).
6. O. Metais and J. R. Herring, J. Fluid Mech., **202**, 117-148 (1989).
7. B. A. Kolovandin, Advances in Heat Transf., **21**, 185-237 (1991).
8. R. Deissler, Int. J. Heat Mass Transf., **6**, 217-270 (1963).
9. D. W. Dunn and W. H. Reid, Heat Transf. in Isotropic Turbulence during the Final Period of Decay, Natl. Advis. Comm. Aeronaut. (1958), TN-4186.
10. V. U. Bondarchuk, B. A. Kolovandin, and O. G. Martynenko, Int. J. Heat Mass Transf., **34**, 11-30 (1991).
11. K. Yoon and Z. Warhaft, J. Fluid Mech., **215**, 601-638 (1990).